

**On Fuzzy Supra b-open map and Fuzzy Supra b-closed map****Mukta Bhatele**

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Abstract—In this paper, we introduce and investigate a new class of sets and maps between fuzzy topological spaces called fuzzy supra b-open sets, fuzzy supra b-open maps and fuzzy supra b-closed maps respectively. Furthermore, we investigate several properties of fuzzy supra b-open maps and fuzzy supra b-closed maps.

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1. INTRODUCTION

The notion of fuzzy sets was introduced by L. A. Zadeh [11] in 1965. C. L. Chang [4] has extended the concept of topology by taking a collection of fuzzy sets instead of crisp sets and developed the theory of Fuzzy Topological spaces. In 1983, A. S. Mashhour et al. [6] introduced the concept of supra topological spaces and studied s-continuous functions and s*- continuous functions. In 2008, Devi et al. [5] introduced supra α -open (closed) sets. In 1987, M. E. Abd El-Monsef et al. [2] introduced fuzzy supra topological spaces and studied fuzzy supracontinuous functions.

In this paper we introduce and study the new class of mappings called fuzzy supra b-open maps and fuzzy supra b-closed maps in

fuzzy topological spaces. Also we define the relation between them.

2. PRELIMINARIES

Let X be a non empty set. A collection τ of fuzzy sets in X is called a fuzzy topology on X if the whole fuzzy set 1 and the empty fuzzy set 0 is the members of τ and τ is closed with respect to any union and finite intersection. The members of τ are called fuzzy open sets and the complement of a fuzzy open set is called fuzzy closed set. The closure of a fuzzy set λ (denoted by $\text{cl}(\lambda)$) is the intersection of all fuzzy closed which contains λ . The interior of a fuzzy set λ (denoted by $\text{int}(\lambda)$) is the union of all fuzzy open subsets of λ . A fuzzy set λ in X is fuzzy open (resp. fuzzy closed) if and only $\text{int}(\lambda) = \lambda$ (resp. $\text{cl}(\lambda) = \lambda$).

Definition 2.1: Let (X, τ) be a fuzzy topological space. A fuzzy set λ in the space X is called:

- (i) Fuzzy semi-open set [1] if $\lambda \leq \text{cl}(\text{int}(\lambda))$ and semi-closed fuzzy set if $\text{int}(\text{cl}(\lambda)) \leq \lambda$.
- (ii) Fuzzy pre-open set [3] if $\lambda \leq \text{int}(\text{cl}(\lambda))$ and pre-closed fuzzy set if $\text{cl}(\text{int}(\lambda)) \leq \lambda$.
- (iii) Fuzzy α -open set [3] if $\lambda \leq \text{int}(\text{cl}(\text{int}(\lambda)))$ and α -closed fuzzy set if $\text{cl}(\text{int}(\text{cl}(\lambda))) \leq \lambda$.

- (iv) Fuzzy supra semi-open set [2] if $\lambda \leq \text{cl}^\mu(\text{int}^\mu(\lambda))$ and fuzzy supra semi-closed set if $\text{int}^\mu(\text{cl}^\mu(\lambda)) \leq \lambda$.
- (v) Fuzzy supra α -open set [2] if $\lambda \leq \text{int}^\mu(\text{cl}^\mu(\text{int}^\mu(\lambda)))$ and fuzzy supra α -closed set if $\text{cl}^\mu(\text{int}^\mu(\text{cl}^\mu(\lambda))) \leq \lambda$.
- (vi) Fuzzy Supra-open \rightarrow Fuzzy Supra α -open \rightarrow Fuzzy supra semi-open \rightarrow Fuzzy Supra b-open

3. FUZZY SUPRA B-OPEN SETS

Definition 3.1: Let (X, μ) be a fuzzy supra topological space. A set λ is called a fuzzy supra b-open (fuzzy supra b-closed) set in $\lambda \leq \text{cl}^\mu(\text{int}^\mu(\lambda)) \cup \text{int}^\mu(\text{cl}^\mu(\lambda))$ (resp. $\text{cl}^\mu(\text{int}^\mu(\lambda)) \cap \text{int}^\mu(\text{cl}^\mu(\lambda)) \leq \lambda$).

Theorem 3.2: Every fuzzy supra semi-open set is fuzzy supra b-open.

Proof: Let λ be a fuzzy supra semi-open set in (X, μ) . Then $\lambda \leq \text{cl}^\mu(\text{int}^\mu(\lambda))$. Hence $\lambda \leq \text{cl}^\mu(\text{int}^\mu(\lambda)) \cup \text{int}^\mu(\text{cl}^\mu(\lambda))$ and λ is fuzzy supra b-closed set in (X, μ) .

The converse of the above theorem need not be true as shown by following example.

Example 3.3: Let (X, μ) be a fuzzy supra topological space, where $X = \{x_1, x_2\}$. Let λ, ϑ be fuzzy sets in X and $\mu = \{0, \lambda, \vartheta, 1\}$ defined as $\lambda(x_1) = 0.2, \lambda(x_2) = 0.3, \vartheta(x_1) = 0.5$ and $\vartheta(x_2) = 0.6$. Here $\gamma(x_1) = 0.4$ and $\gamma(x_2) = 0.5$ is a fuzzy supra b-open set, but it is not fuzzy supra semi-open.

DIAGRAM

Fuzzy Supra-open \rightarrow Fuzzy Supra α -open \rightarrow Fuzzy supra semi-open \rightarrow Fuzzy Supra b-open

Theorem 3.4:

Arbitrary Union of fuzzy supra b-open sets is always fuzzy supra b-open.

Finite intersection of fuzzy supra b-closed sets may fail to be fuzzy supra b-open.

X is a fuzzy supra b-open set.

Proof: (i) Let λ and ϑ be two fuzzy b-open sets. Then, $\lambda \leq \text{cl}^\mu(\text{int}^\mu(\lambda)) \cup \text{int}^\mu(\text{cl}^\mu(\lambda))$ and $\vartheta \leq \text{cl}^\mu(\text{int}^\mu(\vartheta)) \cup \text{int}^\mu(\text{cl}^\mu(\vartheta))$. Then $\lambda \cup \vartheta \leq \text{cl}^\mu(\text{int}^\mu(\lambda \cup \vartheta)) \cup \text{int}^\mu(\text{cl}^\mu(\lambda \cup \vartheta))$. Therefore, $\lambda \cup \vartheta$ is fuzzy supra b-open sets.

(ii) As the definition.

Theorem 3.5: (i) Arbitrary intersection of fuzzy supra b-closed sets is always fuzzy supra b-closed.

(ii) Finite union of fuzzy supra b-closed sets may fail to be fuzzy supra b-closed.

Proof: As above theorem.

Definition 3.6: The fuzzy supra b-closure of a set λ , denoted by $\text{cl}_b^\mu(\lambda)$ is the intersection of fuzzy supra b-closed sets including λ . The supra b-interior of a set λ , denoted by $\text{int}_b^\mu(\lambda)$, is the union of fuzzy supra b-open sets included in λ .

Remark 3.7: It is clear that $\text{int}_b^\mu(\lambda)$ is a fuzzy supra b-open set and $\text{cl}_b^\mu(\lambda)$ is a fuzzy supra b-closed set.

$\lambda \leq \text{cl}_b^\mu(\lambda)$; and $\lambda = \text{cl}_b^\mu(\lambda)$ iff λ is a fuzzy supra b-closed set.

$\text{int}_b^\mu(\lambda) \leq \lambda$; and $\text{int}_b^\mu(\lambda) = \lambda$ iff λ is a fuzzy supra b-open set.

$$\text{cl}_b^\mu(1-\lambda) = 1-\text{int}_b^\mu(\lambda).$$

$$\text{int}_b^\mu(1-\lambda) = 1-\text{cl}_b^\mu(\lambda).$$

Proof: Obvious.

Theorem 3.8: (i) $\text{int}_b^\mu(\lambda) \cup \text{int}_b^\mu(\vartheta) \leq \text{int}_b^\mu(\lambda \cup \vartheta)$;

$$(ii) \text{cl}_b^\mu(\lambda \cap \vartheta) \leq \text{cl}_b^\mu(\lambda) \cap \text{cl}_b^\mu(\vartheta)$$

Proof: obvious.

Proposition: The intersection of a fuzzy supra α -open set and a supra b-open set is a fuzzy supra b-open set.

Fuzzy Supra b-Open Maps and Fuzzy Supra b-Closed Maps

Definition 4.1: A map $f:(X, \tau) \rightarrow (Y, \sigma)$ is called a fuzzy supra b-open (resp. fuzzy b-closed) if the image each open (resp. fuzzy closed) set in X is fuzzy supra b-open (resp. fuzzy supra b-closed) in (Y, ϑ) .

Theorem 4.2: A map $f:(X, \tau) \rightarrow (Y, \sigma)$ is fuzzy supra b-open if and only if $f(\text{int}(\lambda)) \leq \text{int}_b^{\vartheta}(f(\lambda))$ for each set λ in X .

Proof: Suppose that f is a fuzzy supra map. Since $\text{int}(\lambda) \leq \lambda$, then $f(\text{int}(\lambda)) \leq f(\lambda)$. By hypothesis, $f(\text{int}(\lambda))$ is a fuzzy supra b-open set and $\text{int}_b^{\vartheta}(f(\lambda))$ is the largest fuzzy supra b-open set contained in $f(\lambda)$. Hence $f(\text{int}(\lambda)) \leq \text{int}_b^{\vartheta}(f(\lambda))$.

Conversely, suppose λ is an fuzzy open set in X . Then, $f(\text{int}(\lambda)) \leq \text{int}_b^{\vartheta}(f(\lambda))$. Since $\text{int}(\lambda) = \lambda$, then $f(\lambda)$ is a fuzzy supra b-open set in (Y, τ) and f is a fuzzy b-open map.

Theorem 4.3: A map $f:(X, \tau) \rightarrow (Y, \sigma)$ is fuzzy supra b-closed if and only if $\text{cl}_b^{\mu}(f(\lambda)) \leq f(\text{cl}(\lambda))$, then $\text{cl}_b^{\mu}(f(\lambda)) \leq f(\text{cl}(\lambda))$ for each set λ in X .

Proof: Suppose f is fuzzy supra b-closed map. Since for each set λ in X , $\text{cl}(\lambda)$ is fuzzy closed set in X , then $f(\text{cl}(\lambda))$ is a supra b-closed set in Y . Also, since $f(\lambda) \leq f(\text{cl}(\lambda))$, then $\text{cl}_b^{\mu}(f(\lambda)) \leq f(\text{cl}(\lambda))$.

Conversely, let λ be a fuzzy closed set in X . Since $\text{cl}_b^{\mu}(f(\lambda))$ is the smallest fuzzy supra b-closed set containing $f(\lambda)$, then $f(\lambda) \leq \text{cl}_b^{\mu}(f(\lambda)) \leq f(\text{cl}(\lambda)) = f(\lambda)$. Thus, $f(\lambda) = \text{cl}_b^{\mu}(f(\lambda))$. Hence, $f(\lambda)$ is a fuzzy supra b-closed set in Y . Therefore, f is a fuzzy supra b-closed map.

Theorem 4.4: Let (X, τ) , (Y, σ) and (Z, ϑ) be three fuzzy topological space and $f:(X, \tau) \rightarrow (Y, \sigma)$ and $g:(Y, \sigma) \rightarrow (Z, \vartheta)$ be two maps. Then,

If gof is fuzzy supra b-open and f is fuzzy continuous surjective, then g is a fuzzy supra b-open map.

If gof is open and g is fuzzy supra b-continuous injective, then f is a fuzzy supra b-open map.

Proof:(i) Let λ be an open set in Y . Then, $f^{-1}(\lambda)$ is an fuzzy open set in X . Since gof is a fuzzy supra b-open map, then $(\text{gof})(f^{-1}(\lambda)) = g(f(f^{-1}(\lambda))) = g(\lambda)$ (because f is subjective) is a fuzzy supra b-open set in Z . Therefore, g is fuzzy supra b-open map.

Let λ be an fuzzy open set in X . Then, $g(f(\lambda))$ is an fuzzy open set in Z . Therefore, $g^{-1}(g(f(\lambda))) = f(\lambda)$ (because g is injective) is a fuzzy supra b-open set in Y . Hence f is a fuzzy supra b-open map.

Theorem 4.5: Let (X, τ) and (Y, σ) be two fuzzy topological spaces and $f:(X, \tau) \rightarrow (Y, \sigma)$ be a bijective map. Then the following are equivalent:

f is a fuzzy supra b-open map.

f is a fuzzy supra b-closed map.

f^{-1} is a fuzzy supra b-continuous map.

Proof: (i) \Rightarrow (ii) Suppose λ is a fuzzy closed set in X . Then $1-\lambda$ is an fuzzy open set in X and by (i), $f(1-\lambda)$ is a fuzzy supra b-open set in Y . Since f is bijective, then $f(1-\lambda)$

$= 1-f(\lambda)$. Hence, $f(\lambda)$ is a fuzzy supra b-closed set in Y . Therefore, f is a fuzzy supra b-closed map.

(ii) \Rightarrow (iii) Let f is a fuzzy supra b-closed map and λ be fuzzy closed set in X . Since f is bijective then $(f^{-1})^{-1}(\lambda) = f(\lambda)$ which is a fuzzy supra b-closed set in Y . Theorem 7, f is fuzzy supra b-continuous map.

\Rightarrow (i) Let λ be an fuzzy open set in X . Since f^{-1} is a fuzzy supra b-continuous map, then $(f^{-1})^{-1}(\lambda) = f(\lambda)$ is a fuzzy b-open set in Y . Hence, f is fuzzy supra b-open map.

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